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Optimization of "T"-Shaped Fins Geometry Using Constructal Theory and "FEA" Concepts

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ABSTRACT
This paper reports the geometric (constructal) optimization of T-shaped fin assemblies, where the objective is to maximize the global thermal conductance of the assembly, subject to total volume and fin-material constraints. Assemblies of plate fins are considered. It is shown that every geometric feature of the assembly is delivered by the optimization principle and the constraints. These optimal features are reported in dimensionless terms for this entire class of fin assemblies. Based on the constructal theory by Dr. A Bejan, T-shaped fins are developed for better heat conductance as compared to conventional fins. Now the geometry of this T type of fin contains many geometry parameters which affect the overall conductance of the fin. With the same material constraint and volume constraints optimal geometry ratios has been calculated so as to design the fin for its best performance. With focus to the practical situations and heat flow patterns, it is quite complex to calculate the temperatures on a T-shaped fin. It requires the help of FEA concepts and CAE software to optimize the geometry.
Keywords –Conductance, Constructal, Fins, Geometry, Optimization

I. Introduction

Fins are used for thermal conductance of heat from a system to its surrounding for better performance of the system. Conventional fin structures fails in many cases to provide the necessary heat exchange required for a system which results to the poor performance and shorter life of the mechanical or electrical components associated with the system [2]. In replacement of the natural heat exchange process, cooling fluids with forced circulation is adopted in many of the cases. Although effective but circulation of the cooling fluids consumes a lot of energy hence decreasing the overall efficiency of the system. Here comes the need of a new fin geometry subjected to same material and volume constraints.

Constructal theory proposed by Dr. A Bejan develops such geometry for fins which are able to have better conductance than the conventional fins [2] [3]. Constructal theory is the thought that the geometric form visible in natural flow systems is generated by (i.e., it can be deduced from) a single principle that holds the rank of law. The constructal law was first stated for open (or flow) systems: "For a finite-size system to persist in time (to live), it must evolve in such a way that it provides easier access to the imposed currents that flow through it" [1]. This statement has two parts. First, it recognizes the natural tendency of imposed currents to construct shapes, i.e., paths of optimal access through constrained open systems. The second part accounts for the changes in these paths, which occur in an identifiable direction that is aligned with time itself.

The formulation of the constructal law refers to an open system with imposed through flow. If the system is isolated and is initially in a state of internal non equilibrium then it will create optimal geometric paths for passing its internal currents. The constructal law then is the statement that the isolated system selects and optimizes its internal structure to maximize its speed of approach to equilibrium [4]. It provides the answers for many natural evolutions [3].

The constructal method shows us how to minimize geometrically the thermal resistance between a volume and one point, when the total system volume and the volume fraction occupied by high-conductivity "channels" are constrained. The constructal tree or the fin geometry is determined completely from one principle: The minimization of global resistance subject to size constraints. Constructal trees have been determined for heat conduction in two dimensions and for fluid flow through a heterogeneous porous medium [2].

In this paper, process of heat flow through constructal T-shaped fins and their respective equations has been studied. Using those equations, optimum geometry has been designed based on the calculation of different geometric parameters (dimensionless) in order to achieve maximum heat conductance subjected to material and volume constraints.

II. METHODOLOGY

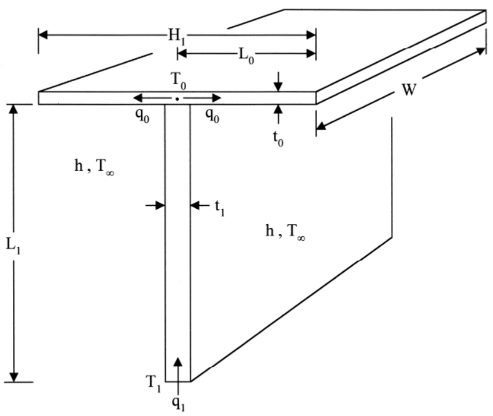

Fig 1: T-shaped assembly of plate fins

Consider the T-shaped assembly of fins sketched in Fig. 1. Two "elemental" fins of thickness t_0 and length L_0 serve as tributaries to a stem of thickness t_1 and length L_1 . The configuration is two-dimensional, with the third dimension (W) sufficiently long in comparison with L_0 andL_1. The heat transfer coefficient *h*is uniform over all the exposed surfaces. Specified are the temperatures of the root (T_1) and the fluid (T_∞). The temperature at the T junction (T_0) is one of the unknowns, and varies with the geometry of the assembly.

The objective of the following analysis is to determine the optimal geometry. (L_1/L_0 , t_1/t_0) that is characterized by the maximum global thermal conductance $q_1/(T_1 - T_\infty)$ where q_1 is the heat current through the root section. The present optimization is subjected to two constraints, namely, the total volume (i.e., frontal area) constraint,

$A = 2L_0\, L_1$//constant// (1)

And the fin-material volume constraint,

$A_f = 2L_0\, t_0 + t_1 L_1$ // constant //(2)

The latter can be expressed as the fin volume fraction $\varphi_1 = A_f/A$, which is a constant considerably smaller than 1.

From [3], for the two elemental fins by using classical assumptions and the solution for a fin withnon-negligible heat transfer through the tip, we havea dimensionless global conductance,

$$\widetilde{q_1} = \frac{q_1}{kW\,(T_1 - T_\infty)} = a\widetilde{t_1}^{1/2}\, \frac{\cosh\left(a\widetilde{L_1}/\widetilde{t_1}^{1/2}\right) - \theta}{\sinh(a\widetilde{L_1}/\widetilde{t_1}^{1/2})} \quad (3)$$

Where,

$$\left(\widetilde{L_0}, \widetilde{t_0}\right) = \frac{(L_0, t_0)}{A^{1/2}} \quad (4)$$

$$a = \left(\frac{2hA^{1/2}}{k}\right)^{1/2} \quad (5)$$

And, $\theta = \frac{T_0 - T_\infty}{T_1 - T_\infty} =$ f($a, \widetilde{L_0}, \widetilde{t_0}\ , \widetilde{L_1}, \widetilde{t_1}$)(6)

The conductance $\widetilde{q_1}$ emerges as a function of $a, \widetilde{L_0}, \widetilde{t_0}\ , \widetilde{L_1}$and $\widetilde{t_1}$: only three of these parameters are free to vary, because of the volume and fin material constraints (1) and (2) [1].

Now a fin with the following geometries is considered for optimization:

A=5000 mm^2
Af= 1000 mm^2
W= 500 mm

Material property assumptions:
Thermal conductivity (K)= .01 w/mK
Film coefficient (*h*)= .00002 w/m^2K

With the considered tabulated values analysis for **T0**&**Tmin**is done by CAE software ANSYS 12.0. Details of the stimulation are as follows:

Element type: Brick 8node 70
Material property: Thermal > Isotropic >Kxx=.01
Mesh: Free mesh with element edge length 5

Loading: 500 degree Celsius at bottom surface and convection at all the other 9 surfaces with film coefficient h=.00002 and bulk temperature= 30 degree Celsius.

III. COMPUTATIONAL ANALYSIS

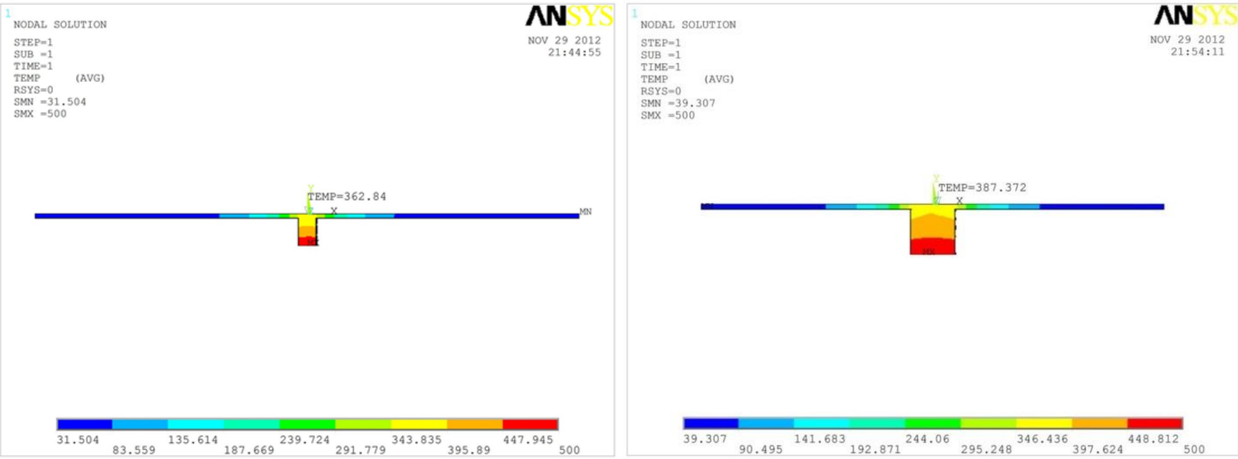

Fig 2: Nodal Temperature Result
For $L_1/L_0 = .1$ and $t_1/t_0 = 4$ (Left) and $L_1/L_0 = .2$ And $t_1/t_0 = 9$ (Right) with $\phi_1 = 0.2$

L1/L0	t1/t0	Tmax	Tmin	T∞	T0	Tmax-T0	Tmax-Tmin	q1
1	2	500	96.384	30	169.418	330.582	403.616	0.1434
0.9	2	500	94.707	30	176.408	323.592	405.293	0.147
0.8	2	500	92.156	30	182.754	317.246	407.844	0.152
0.7	2	500	88.582	30	191.456	308.544	411.418	0.156
0.6	2	500	83.433	30	200.29	299.71	416.567	0.162
	4	500	86.697		258.431	241.569	413.303	0.189
	6	500	82.313		288.091	211.909	417.687	0.196
0.5	2	500	76.338	30	210.315	289.685	423.662	0.166
	4	500	79.308		272.35	227.65	420.692	0.196
	6	500	75.181		300.656	199.344	424.819	0.206
0.4	2	500	67.231	30	223.896	276.104	432.769	0.169
	4	500	69.16		288.38	211.62	430.84	0.202
	6	500	65.767		319.987	180.013	434.233	0.211
0.3	2	500	55.172	30	237.062	262.938	444.828	0.174
	4	500	56.6		306.471	193.529	443.4	0.207
	6	500	54.237		337.329	162.671	445.763	0.219
0.2	2	500	37.068	30	273.871	226.129	462.932	0.162
	4	500	42.308		329.303	170.697	457.692	0.209
	6	500	41.263		361.53	138.47	458.737	0.222
0.1	2	500	31.416	30	291.106	208.894	468.584	0.163
	4	500	31.504		362.89	137.11	468.496	0.195
	6	500	31.35		392.668	107.332	468.65	0.212

Table 1: Different values of **T0**&**Tmin**for different values of **L1/L0**

Now the fins with respective geometries are analysed using the conditions stated above. Tmin and T0 are found from the analysed results and are used to obtain global conductance of the respective geometry.

From Table 1, Maximum value of q1 is found at a value of $L_1/L_0 = .3$ for $t_1/t_0 = 2$ and at $L_1/L_0 = .2$ for $t_1/t_0 = 4$ & 6 both.

As value of q1 for $t_1 / t_0 = 4$ & 6 both are comparatively greater than that of $t_1/t_0 = 2$; hence the optimized value of L_1/L_0 is 0.2.

Now stimulation is done for the optimum value of t_1 / t_0 ; keeping $L_1 / L_0 = .2$ and $\phi_1 = 0.2$; both constant.

t1/t0↓	**q**(L1/L0=0.2)
2	0.162
3	0.199
4	0.2087
5	0.2173
6	0.2221
7	0.225
8	0.2255
9	0.2287
10	0.2272

Table 2: Different values of q for different t_1/t_0

The optimized value of t_1 / t_0 is found to be at $t_1/t_0 = 9$. Now the values of ϕ_1 are varied from .05 to 1 and various values of temperature are stimulated.

The following result is obtained from the above stimulation.

Ø	**q**mm
0.1	0.1776
0.2	0.2285
0.3	0.2551
0.4	0.2557
0.5	0.2889
1	0.3297

Table 3: Different values of conductance **qmm** respective values of **Ø**

It clearly states that, heat transfer rate will increase with increase of area factor.

Parameter	**L1/L0**	**t1/t0**	**φ1**
Optimum value	0.2	9	depends upon size and condition of the system

IV. CONCLUSION

As we find that heat transfer rate will increase with increase of area factor, but occupying space will increase accordingly. Hence a choice has always to be made for area factor depending upon the space available. We have to apply some changes for increasing the robustness of the fin. Geometry should be designed in such a manner that it will possess increased robustness without any significant change in the conductance of the optimized fin. In conclusion, the optimized geometry and performance of T-shaped fins should be optimized for robustness, and can be used to replace conventional fin geometries. Although the fin has been optimized for convection, but hereafter we have to also count for its radiation effects before leading it to practical application.

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